A NOTE ON THE SPATIAL AND TEMPORAL CURRENT COST OF LIVING AND LEVEL OF LIVING INDICES

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1. In this note an attempt is made to develop the notion of current level of living and current cost of living indices. The word current is used to denote current consumption and not the level of wealth possessed and the current cost of living index is its counterpart in prices that is the level of prices of the commodities involved in current consumption. In this sense the normally used 'cost of living index' is the current cost of living index. The cll (current level of living) index should be a suitably chosen function of the commodities consumed and the ccl (current cost of living) index should be a function of the prices of the commodities consumed. The two sets of prices may be either temporal to be used for comparisons over time or spatial to be used for comparison over different regions or towns or centres or spatial-temporal when comparison over both time and space is involved.

2. Let p and q be the price and quantity vectors whose probability-distributions are known with

$$\begin{aligned} E(p) &= \mu \\ E(q) &= \nu \\ E(pp') &= M_p \\ E(qq') &= M_q \end{aligned} \qquad \ldots(1)$$

In fact for the sake of simplicity let us assume that p and q are distributed independently of each other so that

$$E(pq) = E(p)\, E(q) \qquad \ldots(2)$$

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It would not be unrealistic to take the ccl and cll to be linear functions of the prices and quantities respectively. Thus let

$$C = a'p$$
$$L = b'q \qquad \ldots(3)$$

be the ccl and cll indices respectively. a and b are the price-vectors to be estimated so that

$$E(CL - p'q) = 0$$

and $$V(CL - p'q) \text{ is minimized.} \qquad \ldots(4)$$

$p'q$ is the total expenditure.

Equations (4) requires that CL the product of the ccl and cll indices should be equal to the total expenditure in mean and their mean square of deviations from $p'q$ should be the smallest.

Now $$CL - p'q = p'(ab' - I)q.$$

Hence $$E(CL - p'q) = \mu'(ab' - I)\nu = 0 \qquad \ldots(5)$$

and $$S^2 = V(CL - p'q) = E(CL - p'q)^2 \text{ because of (5)}$$
$$= E(p'\,(ab' - I)\,qq'\,(ba' - I)p)$$
$$= T_r(M_p(ab' - I)M_q\,(ba' - I)) \qquad \ldots(6)$$

Hence S^2 has to be minimized w.r.t. a and b subject to the condition (5) and (6).

The required equations are

$$\frac{\partial S^2}{\partial a} - \lambda\partial\,\frac{(\mu'(ab' - I)\nu)}{\partial a} = 0 \qquad \ldots(7)$$
$$\frac{\partial S^2}{\partial b} - \lambda\partial\,\frac{(\mu'(ab' - I)\nu)}{\partial b} = 0$$

and $$\mu'(ab' - I)\nu = 0$$

where λ is the Lagrange multiplier.

After simplification the first two equations reduce to

$$2M_p(ab' - I)M_q b - \lambda\,.\,\mu\nu'\,.\,b = 0$$
$$2M_q(ab' - I)M_p a - \lambda\,.\,\nu\mu'\,.\,a = 0 \qquad \ldots(8)$$

Multiplying the first of these by a' or the second by b' and using the third equation in (7) and simplifying we get

$$2a'M_p(ab' - I)M_q b - \lambda\,.\,\mu'\nu = 0 \qquad \ldots(9)$$

Eliminating λ between (8) and (9) and simplifying we get the $2k$ equations in a and b

$$\left(\frac{\mu a'}{\mu'\nu} - \frac{I}{b'\nu}\right) M_p(ab' - I)M_2 b = 0 \qquad \ldots(10)$$

and $$\left(\frac{\nu b'}{\mu'\nu} - \frac{I}{a'\mu}\right) M_q(ba' - I)M_p a = 0$$

Either equations (8) and (9) or equations (10) can be solved by iterative methods as follows. For solving equations (8) and (9) we first take an initial set of values of a and b, substitute them in (9) and obtain value of λ from (9). Using these values in $ab'-I$, $ba'-I$, $v'b$, $M'a$ (or $M_q b$, $M_p a$ in place of the last two) and λ in equations (8) we solve the resulting linear equations in a and b to get a new set of values of a and b. We proceed with this new set of values of a and b as before and obtain another set of values of a and b and continue with this procedure till stable solutions are obtained. If equations (10) are to be solved then we start as before with an initial set of values of a and b, substitute these for a and b in $(ab'-I)$, $(ba'-I)$, $M_q b$ and $M_q a$ of equations (10) and solve the resulting linear equations in a and b and continue as before with this new set of values and arrive at another set of values of a and b. We continue with this procedure till the solutions stabilize. The ccl and cll indices a_0, b_0 may be called the best unbiased ccl and cll indices.

3. If M_p, M_q, μ and ν are known then the equations above can be solved and a_0 and b_0 obtained. When M_p, M_q, μ and ν are not known a_0, b_0 are not known. Then their estimates have to be obtained from sample information on p and q at a number of spatial-temporal centres. If n sets of independent sample values p_r^1, q_r, $r=1, — —. n$ are available then consistent estimates of a_0 and b_0 can be obtained by replacing population-moments by the corresponding sample moments in equations (8) and (9) or in equations (10) and then the resulting equations can be solved by the same iterative methods as before. Equations (8) and (9) then become

$$\left.\begin{aligned} &2S_p\ (ab'-I)S_q b-\lambda m\ .\ n_1'b=0\\ &2S_q\ (ba'-I)S_p b-\lambda n_1\ .\ m'a=0\\ \text{and}\qquad &2a'S_p\ .\ (ab'-I)\ S_q b-\lambda m'n_1=0 \end{aligned}\right\} \quad \ldots(11)$$

and equations (10) become

$$\left.\begin{aligned} &\left(\frac{ma'}{m'n_1}-\frac{I}{b'n_1}\right) S_p(ab'-I)S_q b=0\\ &\left(\frac{n_1 b'}{m'n_1}-\frac{I}{a'm}\right) S_q(ba'-I)S_p a=0 \end{aligned}\right\} \quad \ldots(12)$$

Here

$$S_p=\frac{1}{n}\sum_u P_{ru}\,P_{su}$$

$$S_q=\frac{1}{n}\sum_u q_{ru}\,q_{su} \qquad r=1,\,-\,-,n$$

$$m_r=\frac{1}{n}\sum_u P_{ru}$$

$$n_{1r}=\frac{1}{n}\sum_{ru} q_{ru}$$

The estimates $\hat{a}$ and $\hat{b}$ thus obtained are also the estimates obtained when

$$\frac{1}{n^2}\sum_r\sum_s (p_r'(ab'-I)q_s)^2\equiv\frac{1}{n^2}\sum_r\sum_s (C_rL_s-q_r'\,p_s)^2 \qquad \text{...(13)}$$

is minimized subject to

$$\frac{1}{n^2}\sum_r\sum_s p_r'(ab'-I)q_s=0$$

The optimization-process in (13) is the sample-counterpart of the optimization-procedure in (6).

The estimates a and b are evidently consistent and hence asymptotically unbiased estimates of a_0 and b_0. If p and q are multivariate normal then m, n_i, S_p, S_q are m.l. estimates of μ, ν, M_p and M_q and hence in general when p, q are multivariate normal, $\hat{a}$, $\hat{b}$ are m.l. estimates of a_0 and b_0 and hence consistent and most efficient estimates of a_0 and b_0. Variances and covariances of these estimates can be worked out and they will be reported later.

4. If

$$\frac{1}{n}\sum_r (p_r'(ab'-I)q_r)^2\equiv\frac{1}{n}\sum_r (C_rL_r-p_r'q_r)^2 \qquad \text{...(14)}$$

is minimized subject to

$$\frac{1}{n}\sum(C_rL_r-p_r'q_r)=0$$

then it can be shown as above that the minimizing equations after simplification are given by

$$2 \sum_{r=1}^{n} b'q_r\, (2\, l_r-\lambda)\, p_r=0$$

$$2 \sum_{r} a'p_r\, (2\, l_r-\lambda)\, q_r=0 \qquad \ldots(15)$$

and

$$\sum_{r} b'\, q_r a'p_r (2\, l_r-\lambda)=0$$

where

$$l_r=\frac{1}{n}\, p_r'\, (ab'-I)\, q_r.$$

These can again be solved by iterative methods as before. Since the n sets of observations are assumed to be independent it can be easily proved that the estimates thus obtained are asymptotically equivalent to a, b above and hence consistent.

5. Once estimates a_0 and b_0 are obtained the ccl and the cll indices for the n centres or for a larger group of centres for which the n centres form a representative group can be worked out and the centres ranked according to these indices. Frequency-distributions, two-way tables, rank and product-moment correlations can be obtained. The rank and product-moment correlations can be tested for significance. The ccl indices may be used for obtaining real wages. The series of ccl indices for different centres thus obtained can be compared with the series of ccl indices obtained with the own quantity-weights and quantity weights of other centres. Note that the series obtained by taking the harmonic mean of the n latter series is the ccl series with average (for the n centre) quantity weights. The ccl series obtained above can be compared with these series also.

6. It has already been mentioned that the centres could be purely temporal or purely spatial or mixed. There are usually not many purely temporal observations available since family-budget surveys are carried out only occasionally and that too with a gap of considerable period when it is felt that the weights could have changed. Hence in such a case the implicit assumption that the a's and b's remain unchanged over time in the above analysis becomes questionable and moreover there would be very few observations to work with. When the centres are spatial, as in the case of middle class or working class family budget surveys in towns in India, the above analysis can be applied. The empirical work on the working class and middle class family-budget surveys in India is being done and will be reported latter. The analysis can also be applied for the same centre when different prices prevail for different income-classes and are available.

In the above analysis it is assumed that p and q are independent. This may not be very realistic. The independence assumption can be relaxed and the estimates obtained as above. The computations however would get complicated since, then

$$E(p_r'(ab'-I)q_r\ q_r'(ba'-I)p_r)$$
$$\neq tr[M_p(ab'-I)M_q(ba'-I)]$$

and
$$E(pr'(ab'-I)qr) \neq \mu'(ab'-I)\nu$$

Fourth order and second order cross-moments of p and q have to be introduced in obtaining a_0 and b_0 and similar sample cross-moments have to be introduced in working out a and b. This will be taken up later.